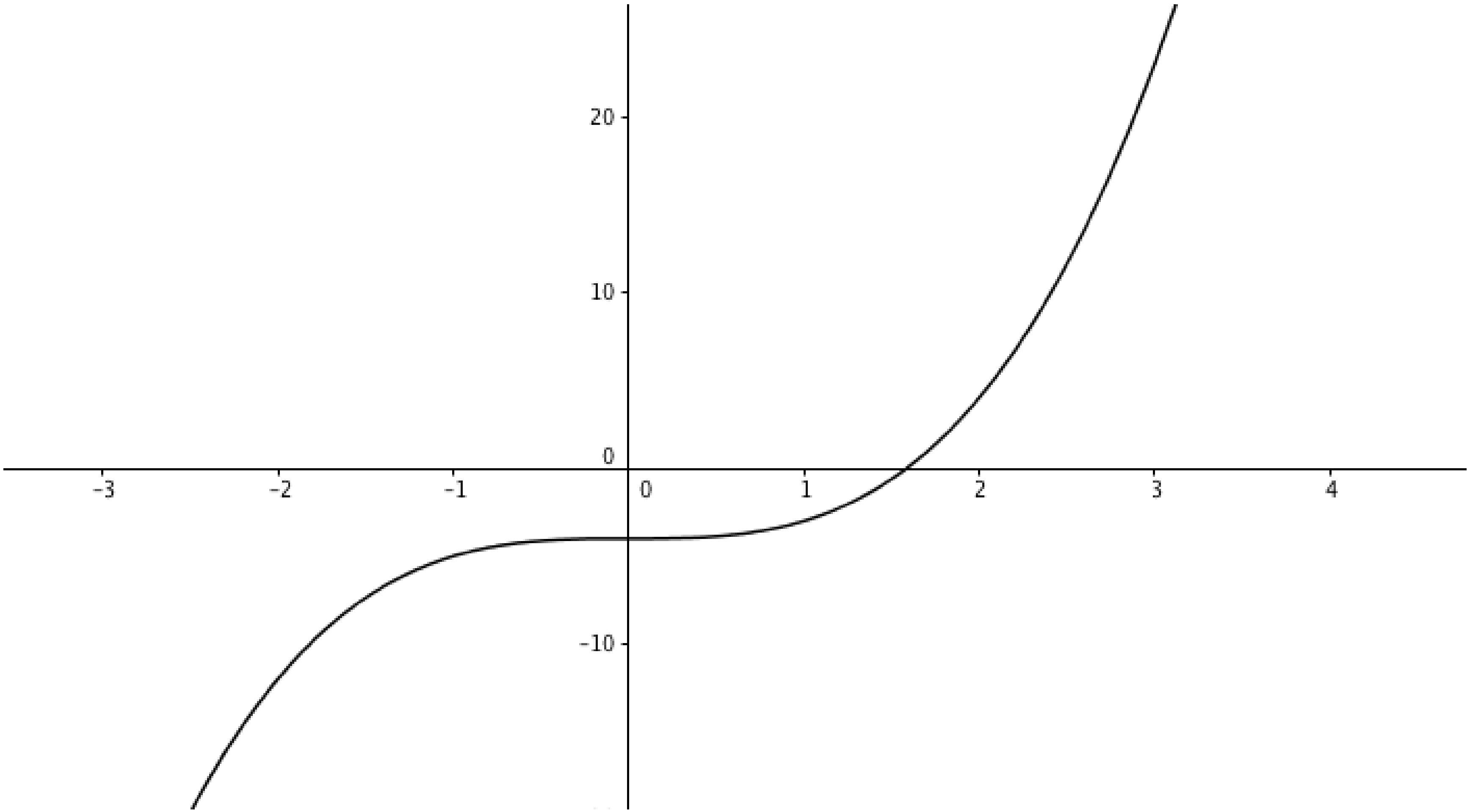
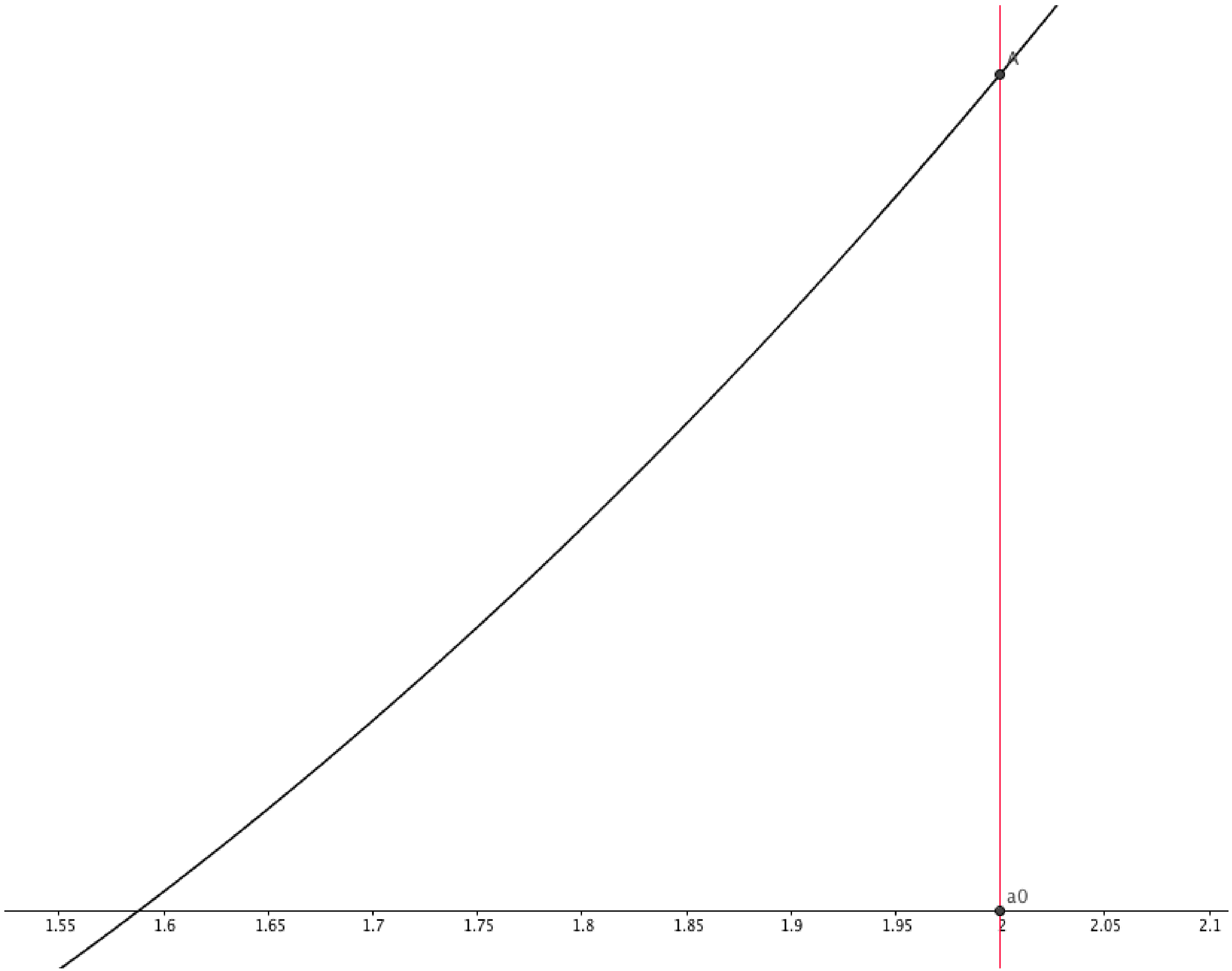
Third degree polynomials

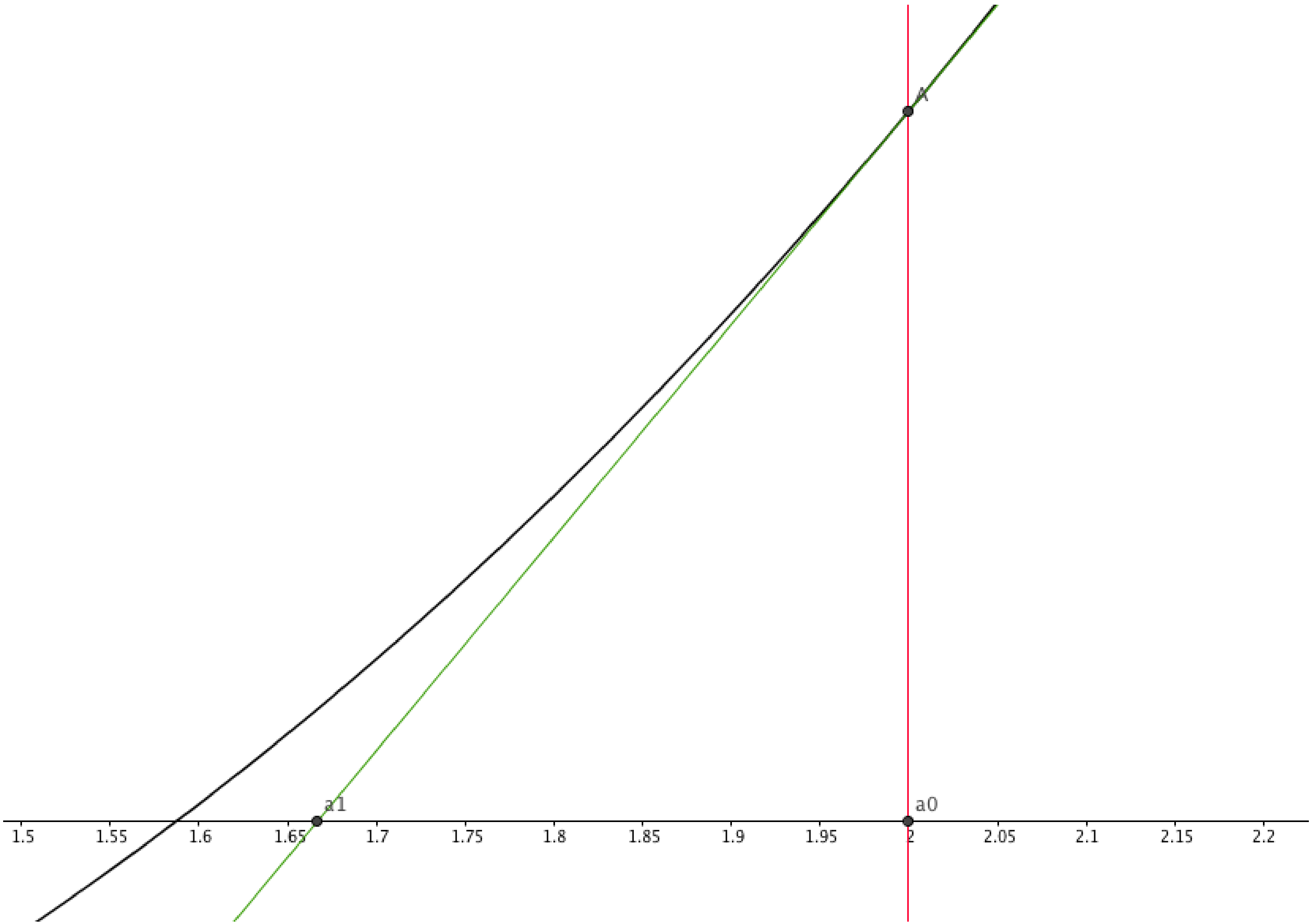
Newton’s Method

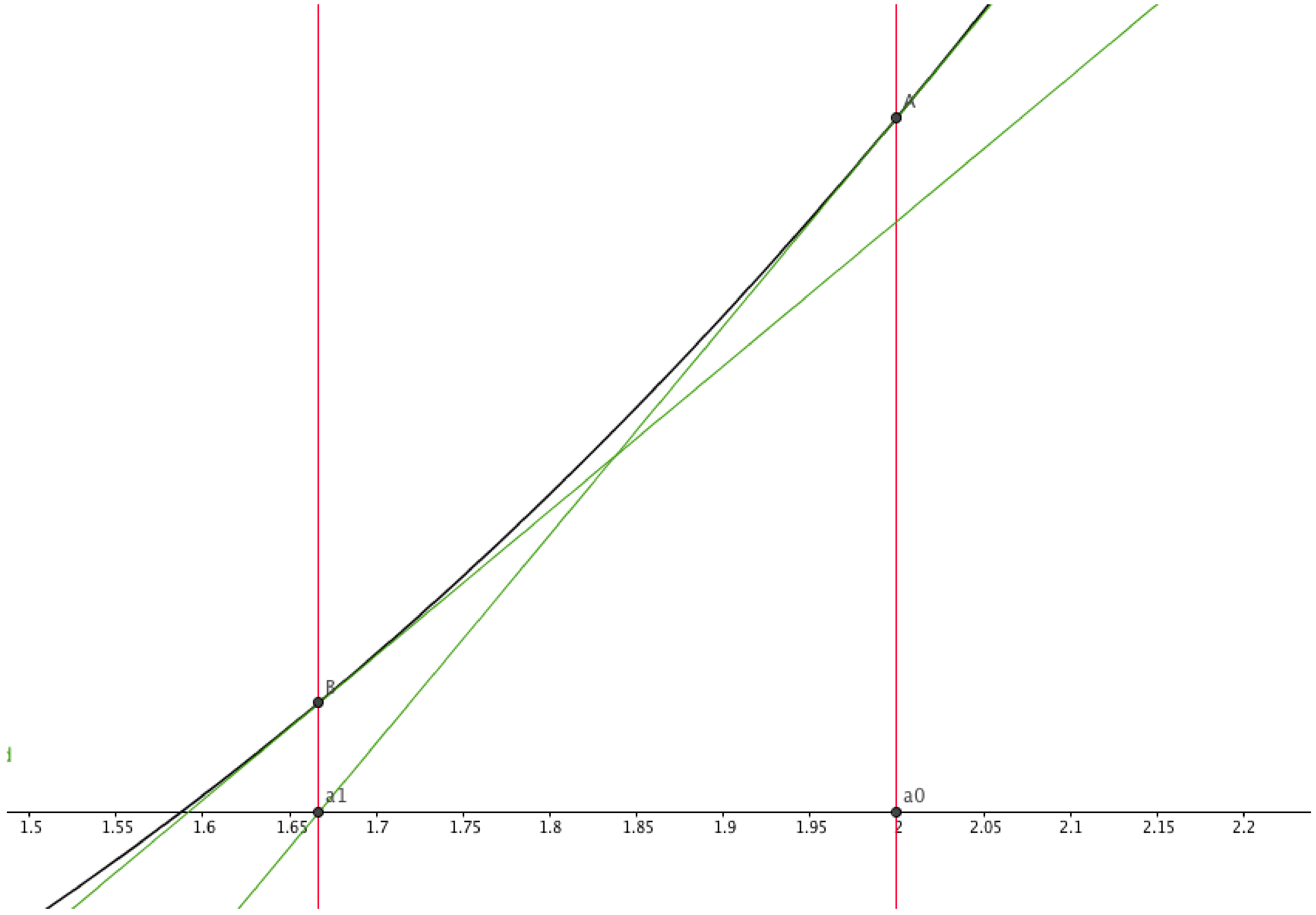
An example: $y=x^3-4$





A
a0
1.55
1.6
1.65
1.7
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1


A
a1
a0
1.5
1.55
1.6
1.65
1.7
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
2.2



A
B
a1
a0
1.5
1.55
1.6
1.65
1.7
1.75
1.8
1.85
1.9
1.95
2
2.05
2.1
2.15
2.2

General form

Generalised form of the x-value of the point closer to the intersection between f(x) and (Ox).

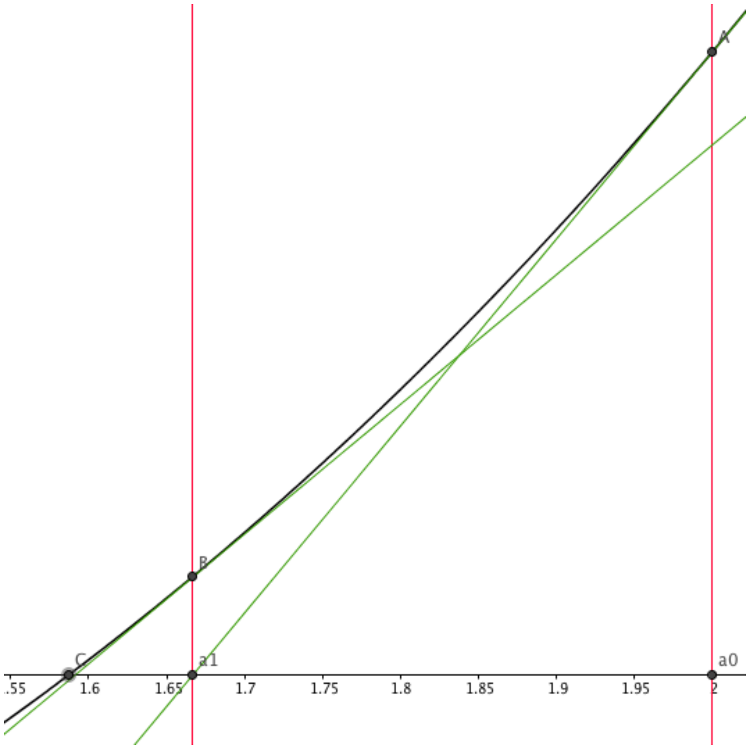
$$y = f'(a) \times (x - a) + f(a)$$

$$y = f'(a_n) \times (x - a_n) + f(a_n)$$

$$a_{n+1} = -\frac{f(a_n)}{f'(a_n)} + a_n$$

Finding the speed of convergence to the intersection of $y=x^3-4$ and (Ox), point M, $x = 1.58740105195$

1. Finding the y-value of A,
2. Calculating the equation of the tangent,
3. Finding point a_1,
4. Tracing a vertical line perpendicular to (Ox) through a_1,
5. Repeating 1-4, then calculate the speed with the points calculated



The values of the points

- The formula for the term a_{n+1}:

$$a_{n+1} = a_n - \frac{(a_n^3 - 4)}{3a_n^2}$$

The values of the five first terms

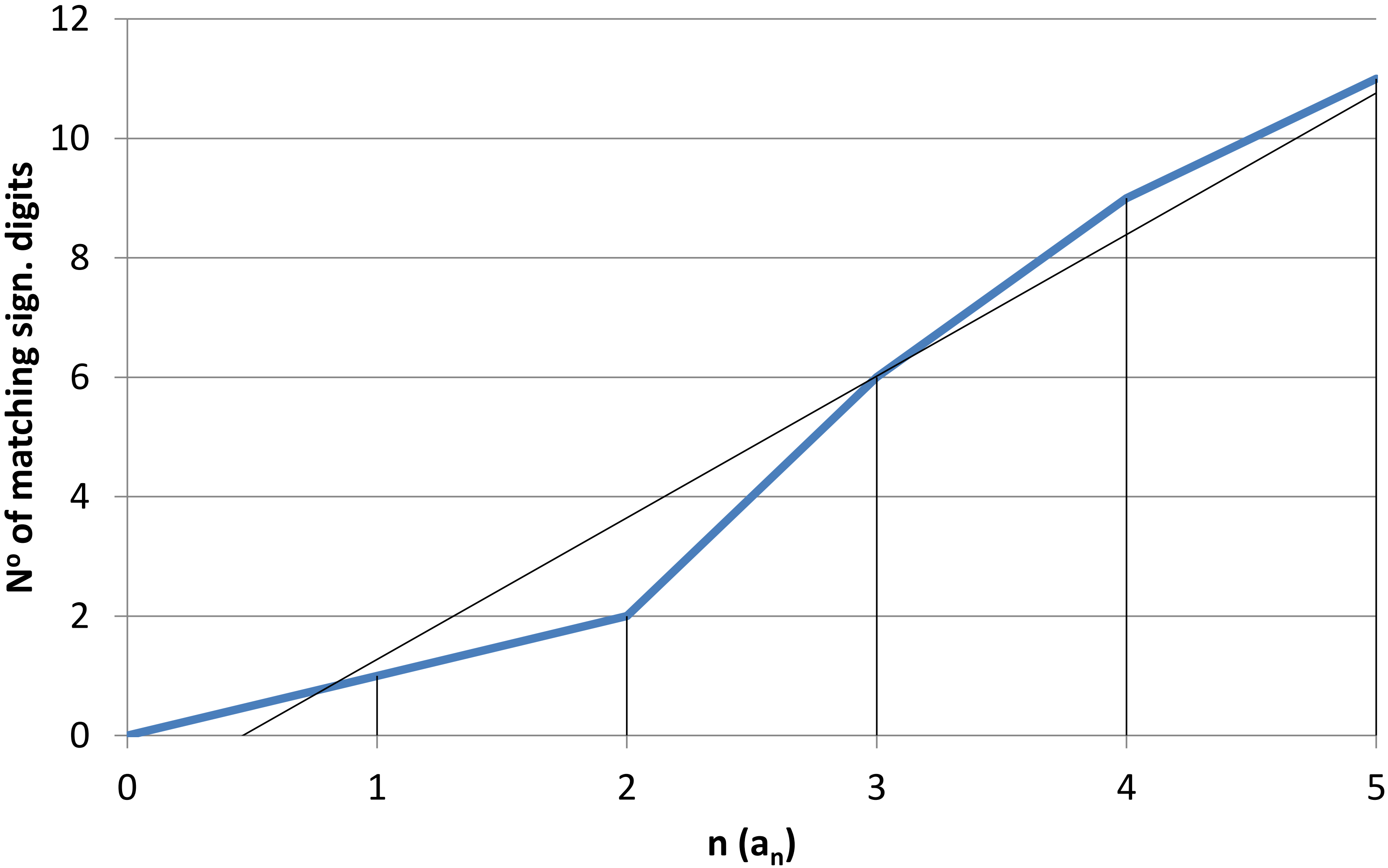
$$a_0 = 2$$
$$a_1 = 1.66666666667$$
$$a_2 = 1.59111111111$$
$$a_3 = 1.58740969614$$
$$a_4 = 1.58740105202$$
$$a_5 = 1.58740105197$$

The number of matching significant digits between M and the approximate points a_n:

$$a_0 - 0$$
$$a_1 - 1$$
$$a_2 - 2$$
$$a_3 - 6$$
$$a_4 - 9$$
$$a_5 - 11$$

The exact solution is: $\sqrt[3]{4} \simeq$ 1,587401051968199474751705639272

Graph of the speed of convergence



Bisection method

- Interval of [1,2]

(Smaller term + larger term)/2

$$\frac{1}{2^n} = 10^{-10}$$

$$n = \frac{10\ln(10)}{\ln(2)}$$

$$n = 34(33.219)$$

To obtain an accuracy of 10 digits, 34 steps are needed with bisection method and only 5 steps with the Newton method.

Thank you